Systems of Inequalities

Whitewater rafting brings thrills and chills for those seeking adventure. Luckily, you can decide just how thrilling and chilling your ride will be based on the river you raft down.

The Playoffs
Graphing Inequalities

LEARNING GOALS

In this lesson, you will:

- Write an inequality in two variables.
- Graph an inequality in two variables.
- Determine which type of line on a graph represents a given inequality.
- Interpret the solutions of inequalities mathematically and contextually.

KEY TERM

- half-plane

Basketball has come a long way to become the fast, action-filled sport it is today. In December 1891, a physical education teacher in Massachusetts was trying to keep his gym class active during the long winter. He wrote down some basic rules, nailed a peach basket to the wall, and basketball was born. At first, basketball was played with an association football, which is similar to a soccer ball. Because association footballs do not bounce very well, dribbling was not part of the game until the 1950s when the basketball we use today was introduced.

Originally, the scoring was pretty simple. If a player got the ball into the basket, their team got one point. The team with the most points at the end of the game won. The first official game was won with a score of 1–0 played on a court that was just half the size of the court basketball is played on today.

Why do you think changes were made to the way the game is played? Do you think basketball would be as popular if changes to the rules and regulations had not been made? Do you think any more changes would ever be made to this or any other sport? Why or why not?

1. Coach Purvis is analyzing the scoring patterns of a few players on his basketball team. Bena has been averaging 20 points per game from scoring on two-point and three-point shots.

    a. If she scores 6 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?

    b. If she scores 7 two-point shots and 2 three-point shots, will Bena meet her points-per-game average?

    c. If she scores 7 two-point shots and 4 three-point shots, will Bena meet her points-per-game average?

2. Write an equation to represent the number of two-point shots and the number of three-point shots that total 20 points.

3. Graph the equation you wrote in Question 2 on the coordinate plane shown.
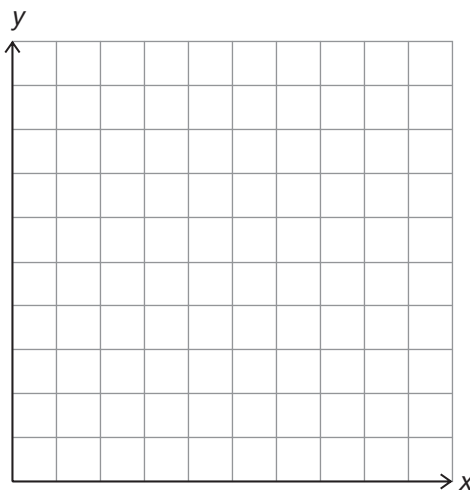
4. Coach Purvis believes that Danvers High School can win the district playoffs if Bena scores at least 20 points-per-game.

    a. How can you rewrite the equation you wrote in Question 2 to represent that Bena must score at least 20 points-per-game?

    b. Write an inequality in two variables that represents this problem situation.

Recall that an inequality is a statement formed by placing an inequality symbol ($<$, $\leq$, $>$, $\geq$) between two expressions. Recall that the forms of linear inequalities in two variables are:

$$ax + by < c$$
$$ax + by \leq c$$
$$ax + by > c$$
$$ax + by \geq c$$

5. Complete the table of values.

Number of Two-Point Shots Scored	Number of Three-Point Shots Scored	Number of Total Points Scored
4	1	
6	1	
7	1	
8	2	
6	4	
9	5	

6. Use the data given in the table to plot the ordered pairs on the graph in Question 3. If the number of total points scored does not exceed Bena's points-per-game average, use an "x" to plot the point. If the number of total points scored meets or exceeds Bena's points-per-game average, use a dot to plot the point.

7. What do you notice about your graph?

7

8. What can you interpret about the solutions of the inequality from the graph?

9. Choose an ordered pair (different from the ordered pairs in the table you completed) located above the graph and an ordered pair that is located below the graph. Does your interpretation of the situation seem correct? Explain your reasoning.

10. Shade the side of the graph that contains the combinations of shots that are greater than or equal to Bena's points-per-game average.

11. How do the solutions of the linear equation $2x + 3y = 20$ differ from the solutions of the linear inequality $2x + 3y \geq 20$?

12. Does the ordered pair (6.5, 5.5) make sense as a solution in the context of this problem situation? Explain why or why not?

PROBLEM 2 Line or Dash? Above or Below?

The graph of a linear inequality is a **half-plane**, or half of a coordinate plane. A line, determined by the inequality, divides the plane into two half-planes and the inequality symbol indicates which half-plane contains all the solutions. These solutions are represented by shading the appropriate half-plane. If the inequality symbol is $\leq$ or $\geq$, the graph is represented by a solid line because the line is part of the solution set. If the symbol is $<$ or $>$, the graph does not include the line and is therefore represented by a dashed line.

1. Determine whether the graph of each inequality would be represented with a solid line or a dashed line on the coordinate plane.

a. $y > 9 - x$

b. $4x - 5y \geq 37$

c. $x + \frac{2}{3}y \leq 6$

d. $x + y < 4$

e. $7x - y > 12$

f. $y \leq 0.65x + 33$

7

2. Consider the linear inequality $y > 4x - 6$. The line that divides the plane is determined by the equation $y = 4x - 6$.

   a. Should the line representing this graph be a solid line or a dashed line? Explain your reasoning.

   b. Graph the inequality on the coordinate plane shown.
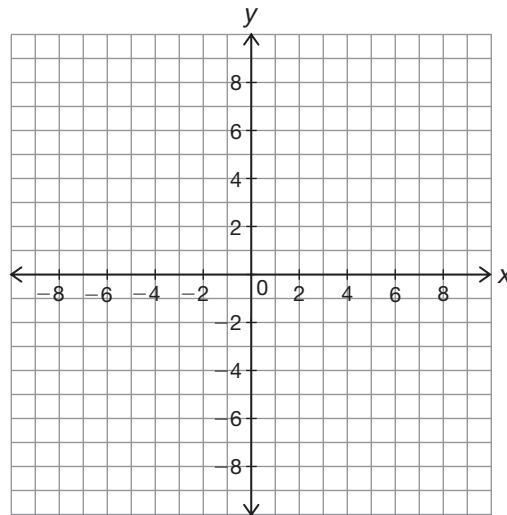
After you graph the inequality with either a solid or a dashed line, you need to decide which half-plane to shade. To make your decision, consider the point (0, 0). If (0, 0) is a solution, then the half-plane that contains (0, 0) contains all the solutions and should be shaded. If (0, 0) is *not* a solution, then the half-plane that does not contain (0, 0) contains all the solutions and should be shaded.

   c. Is (0, 0) a solution? Explain your reasoning.

> It's a good idea to check points in both half-planes to verify your solution.
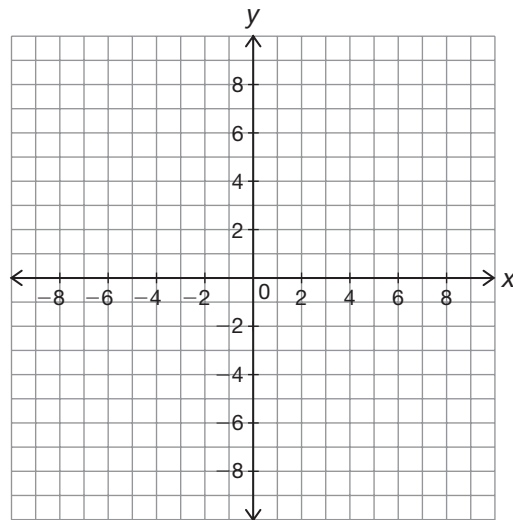
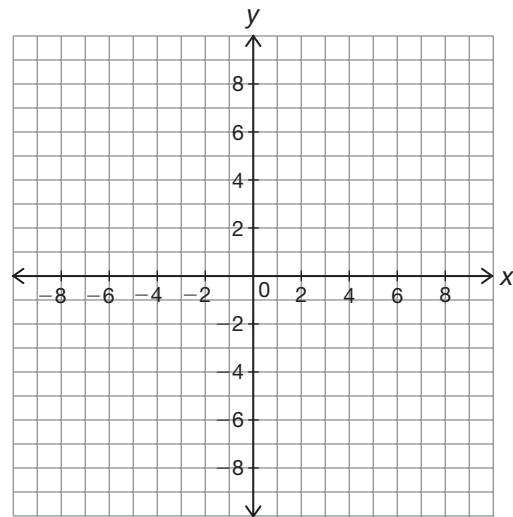   **d.** Shade the correct half-plane on the coordinate plane.

7

3. Graph each linear inequality. Then shade the half-plane that contains the solutions.
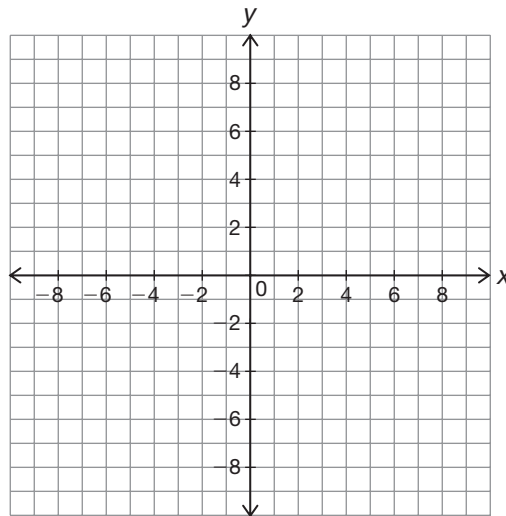
a. $y > x + 3$

Think about the inequality sign and which half-plane will be shaded before you test any points.

b. $y \leq -\dfrac{1}{3}x + 4$

7

c. $2x - y < 4$



PROBLEM 3 I Just Can't Decide!

Your cousin's graduation party is a video game party at the arcade! Each person at the party receives a card with 50 points on it to play the games in the arcade. One of your favorite driving games uses 12 points per game. You also like a basketball game that uses 8 points per game. You want to determine how many times you can play each of these games without exceeding the 50 points on the game card.

1. Write an inequality to represent the problem situation. Define your variables.

2. Complete the table that represents different numbers of times you play the driving game and the basketball game and the total numbers of points used.

Number of Driving Games Played	Number of Basketball Games Played	Total Number of Points Used
0	5	
1	3	
2	3	
2	4	
3	2	
3	3	
4	0	
4	1	

3. Graph the inequality you wrote in Question 1 on the coordinate plane shown. Then use the data in the table to plot the ordered pairs.
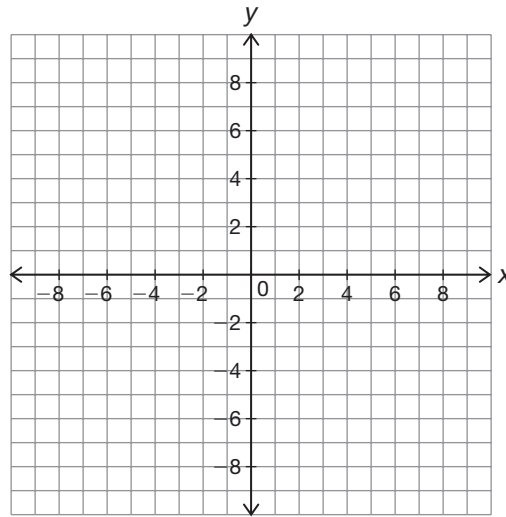
> Again, use an "x" to plot the combinations that exceed the number of points on the card and use a dot to plot the combinations that do not exceed the points on the card.

4. Is the ordered pair $(-1, 8)$ a solution of the inequality you wrote for this problem situation? Why or why not.

5. Is the ordered pair $(7, -3)$ a solution of the inequality you wrote for this problem situation? Why or why not.

6. What can you interpret about the solution set from the graph of this problem situation? Explain your reasoning.

Be prepared to share your solutions and methods.

7

Working the System
Systems of Linear Inequalities

LEARNING GOALS

In this lesson, you will:

- Write and graph systems of linear inequalities.
- Determine solutions to systems of linear inequalities.
- Algebraically prove solutions and non-solutions of systems of linear inequalities.
- Graph systems of linear inequalities using a graphing calculator.

KEY TERMS

- constraints
- solution of a system of linear inequalities

Whitewater rafting is a challenging outdoor activity. It involves navigating through a river or other body of water in an inflatable raft. There are 6 different grades of difficulty in whitewater rafting based on the speed of the current and the hazards rafters may encounter. Grade 1 rafting involves very few rough areas that require some maneuvering of the raft. Grade 1 rafting is good for beginners or children. Grade 6 rafting is so dangerous that there may be times when the waterway is impassable. Rafters can expect to see huge waves and rocks as well as substantial drops. Grade 6 rafting can actually be deadly! However, by using the proper safety gear and traveling with a reliable guide, thousands of people safely enjoy rafting trips every year!

Whitewater rafting often involves a number of people rafting together. Do you think having more or fewer people in the raft would make the trip safer or more dangerous? What else might affect the safety of the raft?

Chase is an experienced whitewater rafter who guides groups of adults and children out on the water for amazing adventures. The super-raft he uses can hold 800 pounds of weight. Any weight greater than 800 pounds will cause the raft to sink, hit more rocks, and maneuver more slowly.

1. Chase estimates the weight of each adult as approximately 200 pounds and the weight of each child under age sixteen as approximately 100 pounds. Chase charges adults $75 and children under age sixteen $50 to ride down the river with him. His goal is to earn at least $150 each rafting trip.

   a. Write an inequality to represent the most weight Chase can carry in terms of rafters. Define your variables.

   Does Chase count when determining the weight and the cost?

   b. Write an inequality to represent the least amount of money Chase wants to collect for each rafting trip.

   c. Write a system of linear inequalities to represent the maximum weight of the raft and the minimum amount of money Chase wants to earn per trip.

In a system of linear inequalities, the inequalities are known as **constraints** because the values of the expressions are "constrained" to lie within a certain region on the graph.

2. Let's consider the past two trips that Chase guides. Determine whether each combination of rafters is a solution of the system of linear inequalities. Then describe the meaning of the solution in terms of this problem situation.
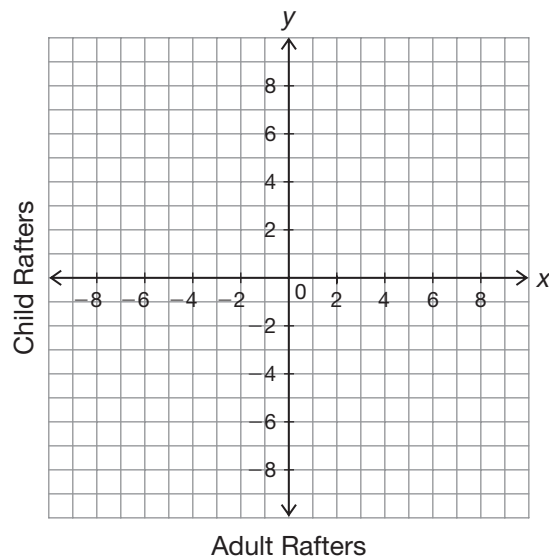
   a. First Trip: Chase guides 2 adults and 2 children.

b. Second Trip: Chase guides 5 adults.

3. Graph the system of linear inequalities on the coordinate plane shown.

Shade the half-plane of each inequality differently. You can use colored pencils or simply vertical and horizontal lines.

The **solution of a system of linear inequalities** is the intersection of the solutions to each inequality. Every point in the intersection region satisfies the solution.

4. Analyze your graph.

a. Describe the possible number of solutions for a system of linear inequalities.

b. Is the intersection point a solution to this system of inequalities? Why or why not?

7

c. Identify three different solutions of the system of linear inequalities you graphed. What do the solutions represent in terms of the problem situation?
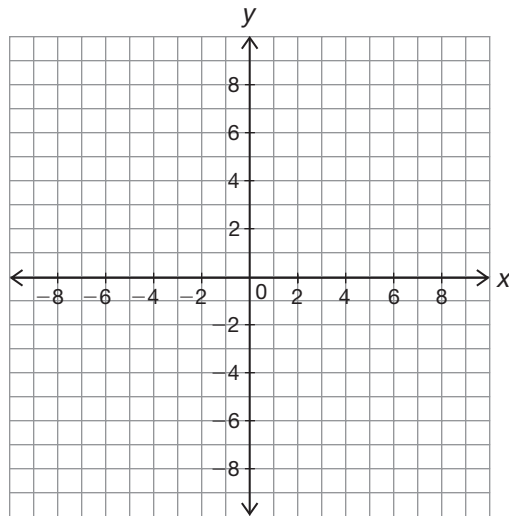
d. Determine one combination of adults and children that is *not* a solution for this system of linear inequalities. Explain your reasoning.

5. Analyze the solution set of the system of linear inequalities shown.

$$\begin{cases} x + y > 1 \\ -x + y \le 3 \end{cases}$$

a. Graph the system of linear inequalities.

Notice the inequality symbols. How do you think this will affect your graph?

b. Choose a point in each shaded region of the graph. Determine whether each point is a solution of the system. Then describe how the shaded region represents the solution.

Point	$x + y > 1$	$-x + y \leq 3$	Description of location
$(-8, 2)$	$-8 + 2 > 1$ $-6 > 1$ ✗	$-(-8) + 2 \leq 3$ $10 \leq 3$ ✗	The point is not a solution to either inequality and it is located in the region that is not shaded by either inequality.

**c.** Alan makes the statement shown.

> 👎 **Alan**
>
> The intersection point is always an algebraic solution to a system of inequalities because that is where the two lines meet.

Explain why Alan's statement is incorrect. Use the intersection point of this system to explain your reasoning.
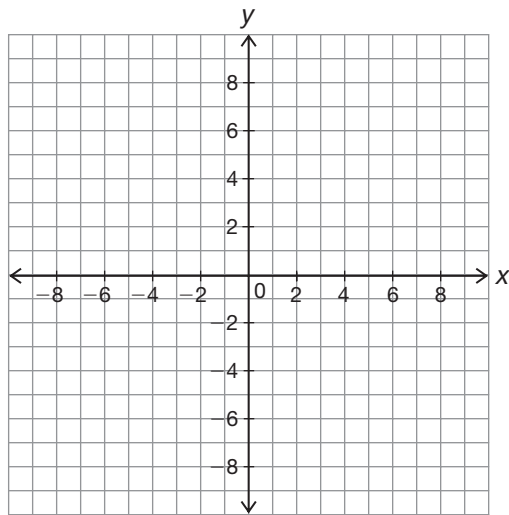
7

6. Solve each system of linear inequalities by graphing the solution set. Then identify two points that are solutions of the system.

a. $\begin{cases} y > 5x + 3 \\ y < 5x - 3 \end{cases}$

b. $\begin{cases} x \geq -4 \\ x \geq 1 \end{cases}$

7
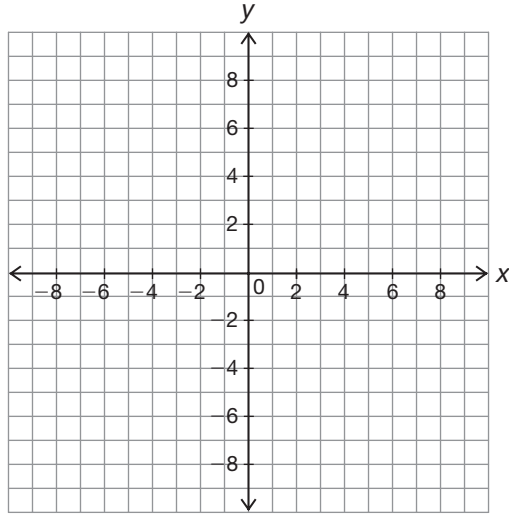
Jackson and a group of friends decide to use the fitness room after school. On the wall, they read the information shown:

Exercise	Calories Burned per Minute
Treadmill—light effort	7.6
Treadmill—vigorous effort	12.4
Stair Stepper—light effort	6.9
Stair Stepper—vigorous effort	10.4
Stationary Bike—light effort	5.5
Stationary Bike—vigorous effort	11.1

Jackson decides to use the stair stepper. He has at most 45 minutes to exercise and he wants to burn at least 400 calories.

1. Write a system of linear inequalities to represent Jackson's workout. Define your variables.

7

Let's graph the system you wrote in Question 1.

You can use a graphing calculator to graph a system of linear inequalities.

Step 1: Press **Y=** and enter the two inequalities as Y_1 and Y_2.

Remember to solve for the y-value before entering the inequalities.

Step 2: While still in the **Y=** window, access the inequality function by moving your cursor to the left until the \ flashes. Press **ENTER** to select the appropriate inequality symbol (◣ or ◢).

When choosing the inequality symbol, think about the half-plane you must shade.

Step 3: Press **WINDOW** and set the bounds.

Set the WINDOW for this problem using the bounds [0, 50] X [0, 50].

Step 4: Press **GRAPH**.

2. Graph the system of inequalities from Question 1 on the coordinate plane shown. Be sure to label your axes.
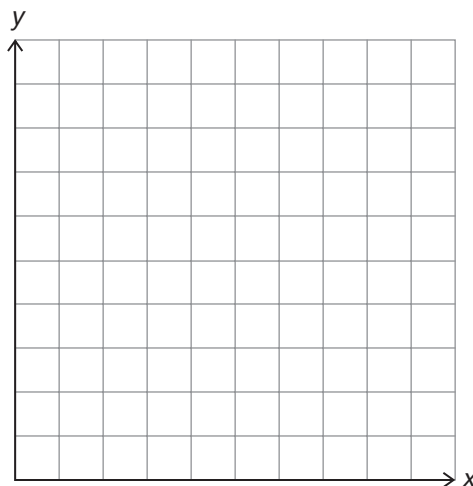
3. Analyze your graph.

  a. Identify two different solutions of the system of inequalities using the **value** function of your graphing calculator.

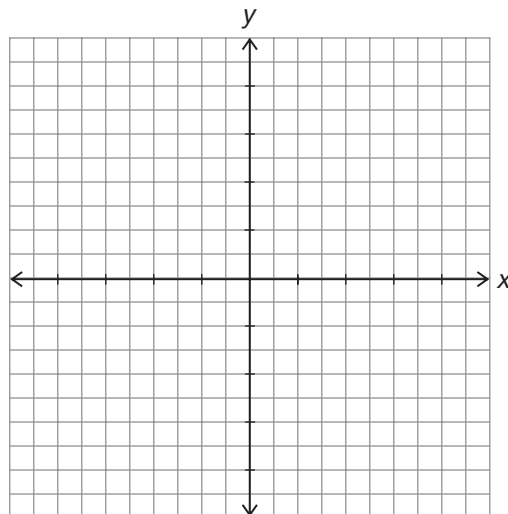 **b.** Interpret your solutions in terms of Jackson's workout.

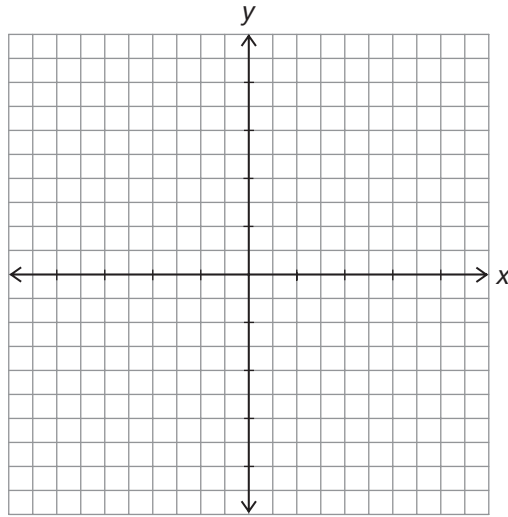 **c.** Algebraically prove that your solutions satisfy the system of linear inequalities.

4. Solve each system of linear inequalities using your graphing calculator. Graph each system then identify two points that are solutions to the system on the grid shown.

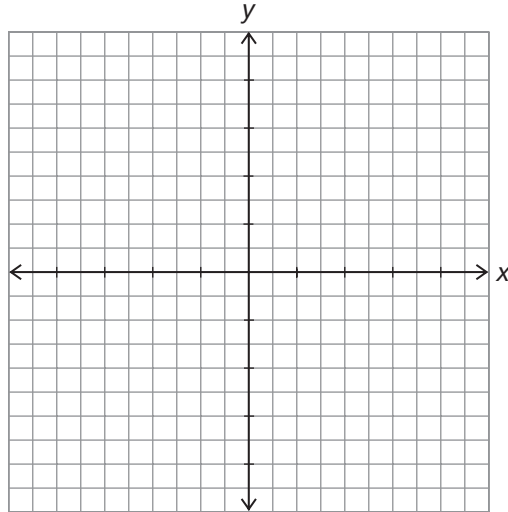 **a.** $\begin{cases} y < \frac{3}{5}x + 3 \\ y > -\frac{3}{5}x + 3 \end{cases}$

7

b. $\begin{cases} y > \dfrac{3}{5}x + 3 \\ y < -\dfrac{3}{5}x + 3 \end{cases}$

c. $\begin{cases} y > \dfrac{3}{5}x + 3 \\ y > -\dfrac{3}{5}x + 3 \end{cases}$

7

d. $\begin{cases} y < \dfrac{3}{5}x + 3 \\ y < -\dfrac{3}{5}x + 3 \end{cases}$
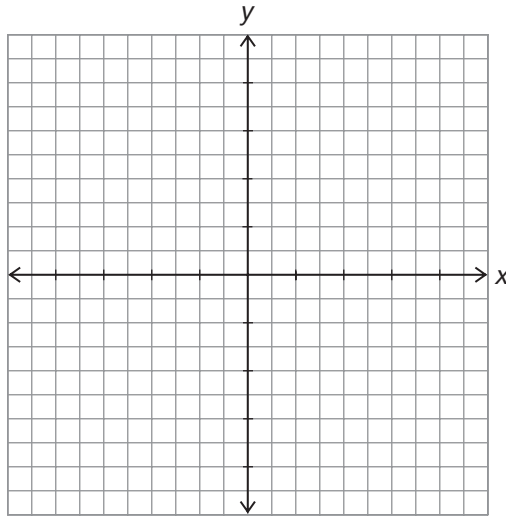
5. Adele states that since the equations in each system for Question 4 are the same, the graphs and solutions should all be identical. Is Adele's statement true? Explain your reasoning.

Be prepared to share your solutions and methods.

7

7

Our Biggest Sale of the Season!

Systems with More Than Two Linear Inequalities

LEARNING GOALS

In this lesson, you will:

- Solve systems of linear inequalities.
- Maximize linear expressions on a region in the coordinate plane.

You've probably heard the sales pitches on the radio or seen the advertisements on television—you've probably even seen them on-line. Yes, it's the biggest sale of the season! Black Friday is here! This Friday in November has been the busiest shopping day of the year for at least a decade and that trend does not seem to be changing any time soon. Black Friday is named as such because this tends to be the day on which retailers begin to turn a profit for the year, which is known as "being in the black." But how can a day known for huge blowout sales be a day when retailers begin making a profit? Due to the extended store hours (some stores open as early as midnight!) and the great deals, the stores tend to see more sales that usual. So even though they have cut their prices, so much more is sold that it makes up for the reduced prices.

Do you think Black Friday deals are worth the crowds and stress that come along with the sales? How can you determine if a sale is as good as it sounds?

Miguel's eye doctor informed him that he needs glasses. However, Miguel shouldn't fret as *I've Got My Eye-Glasses On You* is having a sale on all eyeglass frames. The advertisement in the window is as shown:

> **Save 60% to 75% On All Frames**
> **Regularly Priced at**
> **$120 − $360**

Previously, you solved a system containing two linear inequalities. However, systems can consist of more than two linear inequalities.

1. Use the advertisement to write two inequalities that represent the regular price of eyeglass frames. Let r represent the regular price of the frames.

2. Use the same advertisement to write two inequalities that represent the reduced price of the eyeglass frames. Let s represent the sales price of the frames in terms of r.

> When an item is 20% off the regular price, you can think of that item costing 80% of the regular price!

3. Heather wrote this system of linear inequalities for the problem situation.

> 👎 **Heather**
>
> $$\begin{cases} r \geq 120 \\ r \leq 360 \\ s \leq 0.6r \\ s \geq 0.75r \end{cases}$$

Explain why Heather's system of linear inequalities in incorrect.

4. Graph each inequality on the grid shown. Represent the regular price of the glasses as the *x*-axis and the sale price as the *y*-axis. Use the information in the table shown to set the bounds and intervals for your graph. Do not forget to add a unit of measure for each axis label.

Variable Quantity	Lower Bound	Upper Bound	Interval
Regular Price	0	400	40
Sale Price	0	200	20
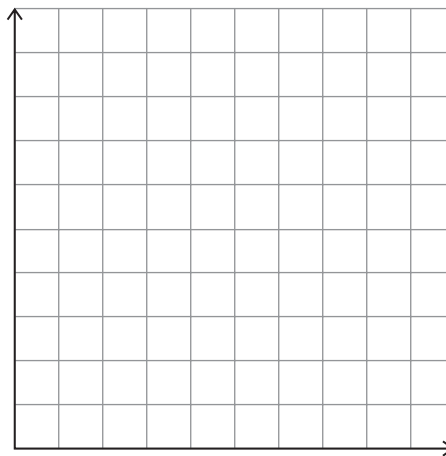
When graphing a system of linear inequalities, you must determine the portion of the graph that satisfies *all* the inequalities in the system.

5. Shade the portion of the graph that satisfies the system of linear inequalities. What shape does the solution region resemble?

6. Using your graph, about how much would Miguel expect to spend if he purchases eyeglasses that are regularly priced at $320?

7

7. Miguel is definitely going to purchase a pair of eyeglasses that are on sale. What is the least amount of money Miguel can expect to spend? What is the greatest amount he can expect to spend?

8. Miguel decides on a pair of eyeglasses that are regularly priced at $240.

> The graph shows me the sale price of the eyeglasses, but how can I determine how much he will save?

   a. Can Miguel expect to save more or less than $140 off the purchase price of this pair of eyeglasses? Use your graph to determine an approximate answer.

   b. Use algebra to determine the greatest amount of money Miguel can save by purchasing eyeglasses that are regularly priced at $240.

7

PROBLEM 2 Give It Up For Macho Nacho

Maize Tortillas and Chips is introducing a new chip to their product line. The Macho Nacho claims to hold six times more cheese than any other tortilla chip on the market. However, the release of this monumental chip couldn't have come at a worse time. The failure of the Brussels sprout chip in 2011 has caused upper management to cut advertising and marketing budgets for the upcoming year. The vice president of Maize has announced that *all* advertising budgets less than $15,000 will be cut by a range of 10% to 32% in 2012.

1. Write a system of inequalities to represent this problem situation. Let *a* represent the advertising budget for 2011, and let *c* represent the advertising budget for 2012.

2. Graph each inequality on the grid. Represent the advertising budget for 2011 on the *x*-axis and the advertising budget for 2012 on the *y*-axis. Do not forget add labels and units of measure for each axis.
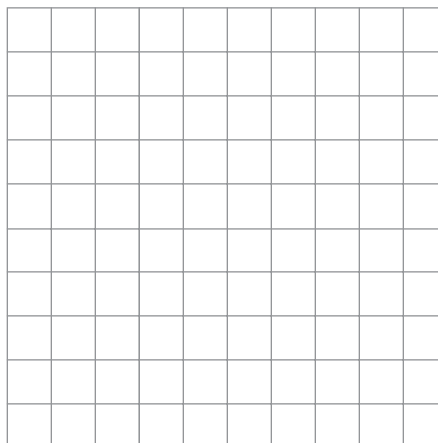
3. Use algebra to determine the points of intersection for the solution region on the coordinate plane. Then label these points on your graph.

4. Predict the point of the solution region that will have the least value. Explain why this point has the least value.

5. Terrance claims that the greatest value in the solution region that satisfies the system is (15,000, 13,500). Shira thinks that Terrance is incorrect and claims that this point does *not* satisfy the system. Who is correct? Explain to the student who is incorrect how to correct his or her claim.

6. Jorge makes a claim about the shape of the solution region.

👍 **Jorge**

The solution region of the system is a triangle.

How can you prove that Jorge is correct?

See! Systems With More Than Two Inequalities *Aren't* Scary!

As you saw in Problems 1 and 2, inequalities can be helpful when determining a solution set for a particular system of linear inequalities. The steps you should remember when solving a system of linear inequalities with more than two inequalities are:

- Graph each linear inequality.

- Determine and shade the region that contains the solution sets that satisfy all the inequalities in the system.

- Determine all the points of intersection for the boundary lines that make the vertices of the solution region.

- Remember to use a closed point if the point is part of the solution region, and use an open dot if the point is not part of the solution region, but is a point of intersection.

1. Graph the solution set for each system of linear inequalities. Label all points of intersection of the boundary lines. Then determine a point the satisfies all the linear inequalities in the system.

a. $\begin{cases} x \geq 0 \\ y \geq 0 \\ x + y \leq 6 \\ y \geq 2x - 5 \end{cases}$

7
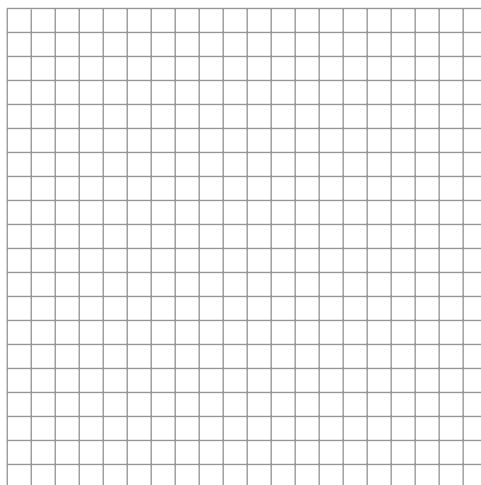
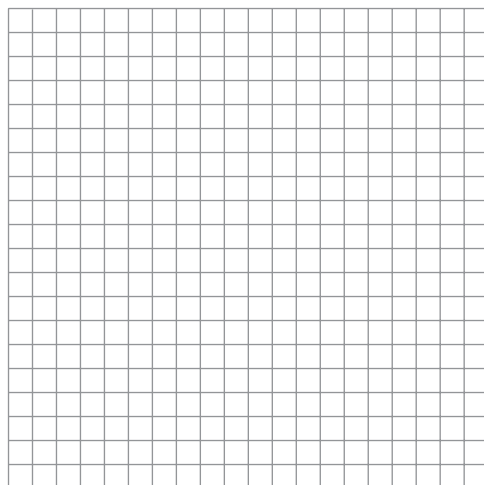**b.** $\begin{cases} x \geq 0 \\ y \geq 0 \\ y \leq -2x + 8 \end{cases}$
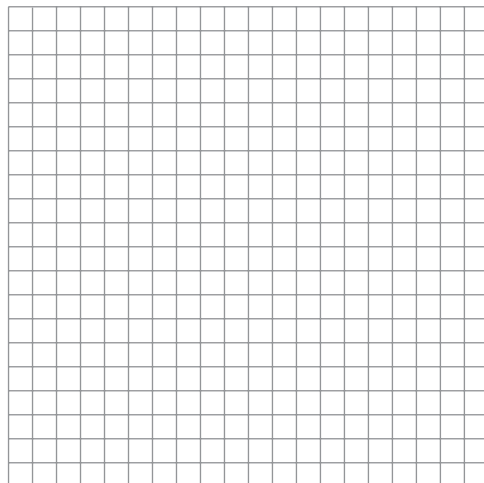
c. $\begin{cases} y < 2x + 5 \\ x + y < 3 \\ 2x - 3y \leq 6 \end{cases}$

Is the point (2, 1) a solution of the system? Why or why not? Use algebra to support your answer.

7

Be prepared to share your solutions and methods.

Take It to the Max . . . or Min
Linear Programming

LEARNING GOALS

In this lesson, you will:

- Write systems of inequalities with more than two inequalities.
- Determine constraints from a problem situation.
- Graph systems of linear inequalities and determine the solution set.
- Identify the maximum and minimum values of a linear expression.

KEY TERM

- linear programming

Have you ever wondered why some products cost what they do? How does a company determine how much to charge for an item? The formula for determining the cost of an item takes into account many different factors. Some things to consider are the cost of materials to make the product, the cost of labor to make the product, the cost of transporting the product, and of course, the hope of making a profit on the product so that there is more money for new products. However, sometimes even these factors may not be the answer to the perfect cost versus profit. Companies also have to pay attention to how other companies set the price for similar products, whether this item is a big seller, and how much consumers are willing to pay for the product. It takes a lot of information and analysis to determine how much to charge for a product.

Do you think there are ever times when a company sells an item for less than it is worth? When might this happen? Do you think this is a good business strategy?

A company, TVs4U, makes and sells two different television models: the HD Big View and the MegaTeleBox.

- The HD Big View takes 2 person-hours to make and the MegaTeleBox takes 3 person-hours to make.

- TVs4U has 24 employees, each working 8 hours a day, which is equivalent to 192 person-hours per day.

- TVs4U's total manufacturing capacity is 72 televisions per day.

- TVs4U cannot make a negative number of televisions.

   1. Define variables to represent the number of each model television produced.

Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities.

   2. Write a system of inequalities to represent the constraints of this problem situation.

   3. Graph the system of inequalities on the coordinate plane shown. Shade the region that represents the solution set.

Number of MegaTeleBoxes Produced (y-axis, 10 to 90)
Number of HD Big Views Produced (x-axis, 0 to 90)

7
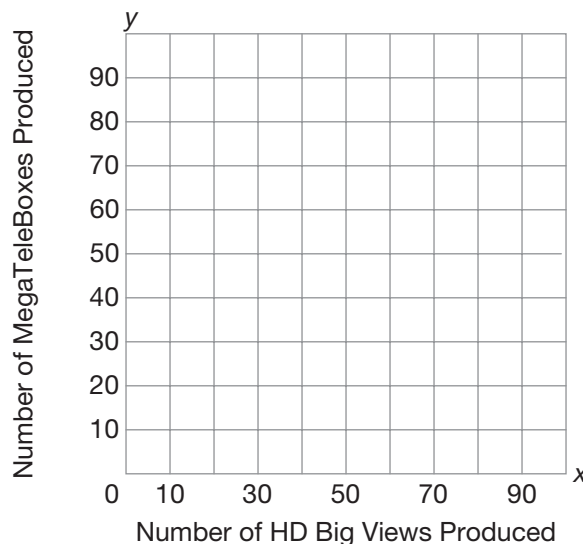
Many companies and businesses are interested in determining when they are maximizing or minimizing their profit or costs. The maximum and minimum values of a system of inequalities occur at a vertex of the region defined by the system.

4. Label all intersection points of the boundary lines.

To determine the maximum and minimum values, you must substitute the coordinates of each point into a given function.

Let's say TVs4U sells the HD Big View for $175 and sells the MegaTeleBox for $205. They want to determine how many of each television they should make and sell to maximize their profits.

Write the function for the given problem situation.

$$P(b, m) = 175b + 205m$$

Insert the coordinates of each intersection point of the system.

$$P(0, 0) = 175(0) + 205(0) = 0$$

$$P(0, 64) = 175(0) + 205(64) = 13{,}120$$

$$P(24, 48) = 175(24) + 205(48) = 14{,}040$$

$$P(72, 0) = 175(72) + 205(0) = 12{,}600$$

The maximum profit is represented by the number of televisions made and sold that results in the greatest number.

5. How many of each television should TVs4U produce to earn the maximum profit? Explain your reasoning.

7

6. TVs4U is trying to determine the price of each model of television. For each set of prices, determine how many of each model should be made to maximize the profit. Then determine the maximum profit. Assume all televisions that are made are sold.

    a. The price of the HD Big View is $250 and the price of the MegaTeleBox is $300.

    b. The price of the HD Big View is $250 and the price of the MegaTeleBox is $375.

c. TVs4U's boss, Mr. Corazon, sends out a memo with his ideas on maximizing the company's profit.

> **Mr. Corazon**
>
> Obviously, we will make the most money by only making and selling the most expensive television model. Therefore, we should focus on producing and selling 72 MegaTeleBoxes each day for $375 a piece.

Explain why Mr. Corazon's idea is incorrect.

## PROBLEM 2 Cashing in on Cell Phones

The cell phone company, Speed of Sound (SOS), produces two types of cell phones. The SOS Smartcall has advanced download speeds and capability which the SOS Basic does not. The assembly lines can produce at most a total of 180 cell phones each day and the company always has at least 40 of each type of cell phone produced and ready for shipping. One SOS Smartcall requires 3 person-hours and $75 of materials to produce. One SOS Basic requires 4 person-hours and $60 of materials to produce. The company has 640 person-hours of labor available daily. The company has budgeted $12,900 for the cost of materials each day.

1. Define your variables and identify the constraints as a system of linear inequalities.

7

2. Graph the solution set for the system of linear inequalities on the coordinate plane shown. Label all intersection points of the boundary lines.
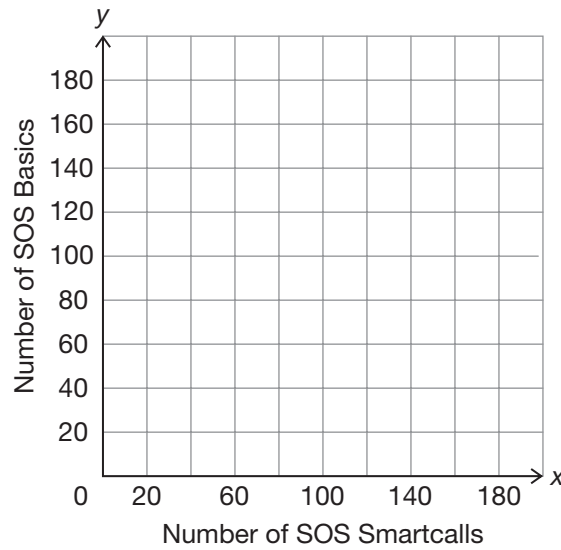
3. The profit from the Smartcall is $30 and the profit from the Basic is $35.

　a. Write a function to represent the total profit.

　b. Paige states that this problem is unrealistic because no one would ever sell a really good cell phone for only $30. Is Paige's statement correct? Why or why not?

4. How many of each type of cell phone should the company produce and sell to maximize its profit? Determine the maximum profit.

5. A competitor has reduced the price of its advanced capability cell phone. In order to compete, SOS will have to decrease its profit on the Smartcall to $25. How will this affect the number of cell phones SOS needs to produce and sell in order to maximize its profit?

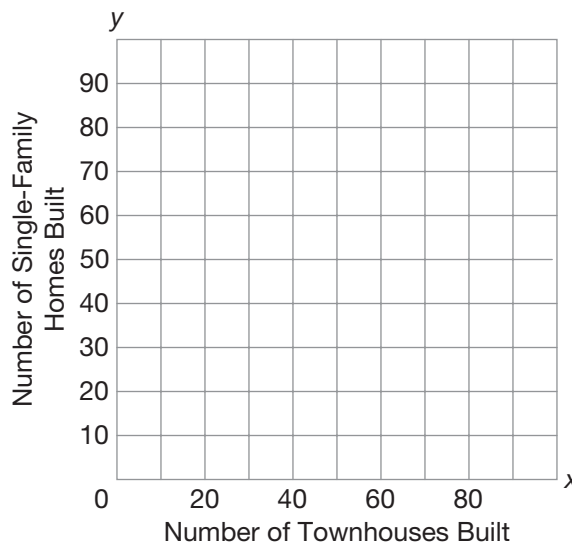## PROBLEM 3 Planning a Housing Development

A building developer is planning a new housing development. He plans to build two types of houses: townhouses and single-family homes. The developer bought a plot of land to build on which already has 20 townhouses and 10 single-family homes built on it. The plot of land has room for the developer to build 100 more homes. It takes the workers 2 months to build a townhouse and 3 months to build a single family home. The developer wants this development complete in 20 years.

1. Define your variables and identify the constraints as a system of linear equations.

2. Graph the solution set for the system of linear inequalities on the coordinate plane shown. Label all intersection points of the boundary lines.

Remember, when comparing or calculating, you must use the same units of measure.

Number of Single-Family Homes Built (y-axis): 10, 20, 30, 40, 50, 60, 70, 80, 90

Number of Townhouses Built (x-axis): 0, 20, 40, 60, 80

7

3. It costs the developer $300,000 to build each townhouse and $450,000 to build each single-family home. Just as the project begins the developer realizes the housing market is not good. He is worried he will not be able to sell all his homes and wants to save his money, but he still needs to build the homes. How many of each type of home should he build if he wants to minimize his costs while still completing the development?

4. The developer sells the houses himself. He sells each townhouse for $325,000 and each single-family home for $490,000.

    a. How much profit does he make by selling each type of house?

    b. Write a function to represent the profit the developer makes for selling a certain number of each type of home.

    c. If the developer uses the plan worked out in Question 3 to minimize his costs, will he maximize his profit? Explain your reasoning.

7

Be prepared to share your solutions and methods.

KEY TERMS

- half-plane (7.1)
- constraints (7.2)

- solution of a system of linear inequalities (7.2)
- linear programming (7.4)

7.1 Writing a Linear Inequality in Two Variables

An inequality is a statement that is formed by placing an inequality symbol ($<, >, \leq, \geq$) between two expressions. Inequalities with two unknowns can be written using data from a problem situation.

Example

Susan is buying a new bike. Her uncle is giving her $25 to put toward the bike, and she's going to use part of her savings to pay for the rest. However, she does not want to use any more than 75% of her savings.

Let x represent the amount of money in Susan's savings and let y represent the cost of the bike in dollars.

$y \leq 0.75x + 25$

### **7.1** Graphing a Linear Inequality in Two Variables

The graph of a linear inequality is a half-plane, or half of a coordinate plane. A line, determined by replacing the inequality symbol with an equals sign, divides the plane into two half-planes. The inequality symbol identifies which half-plane contains all the solutions. If the symbol is $\leq$ or $\geq$, the line of the graph is part of the solution and is solid. If the symbol is $<$ or $>$, the line of the graph is not part of the solution and is represented with a dashed line. Use (0, 0) as a test point to determine which half of the plane is the solution of the inequality and should therefore be shaded.

Example

$y < 5x - 2$

$0 < 5(0) - 2$

$0 < -2$ ✗

447
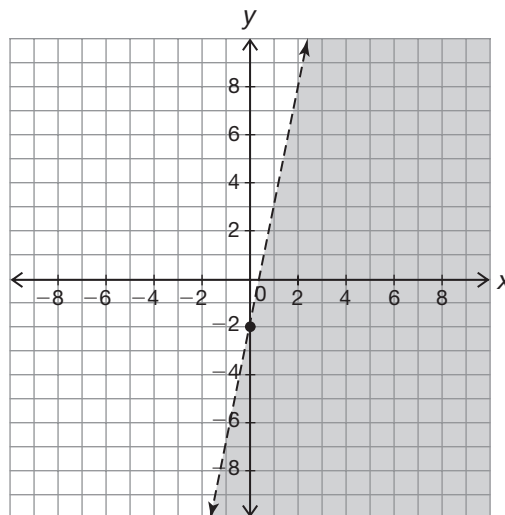
Writing a System of Linear Inequalities

An inequality is a statement that is formed by placing an inequality symbol ($<$, $>$, $\leq$, $\geq$) between two expressions. Two related inequalities can be written using the data from a problem situation.

Example

John is selling popcorn and trail mix to raise money for a school trip. The popcorn sells for $3 a bag, and the trail mix sells for $2 a bag. John needs to raise at least $250 for his trip. Also, if he sells at least 100 items, he gets a special prize.

Let x represent the number of bags of popcorn John needs to sell and let y represent the number of bags of trail mix John needs to sell.

$3x + 2y \geq 250$ and $x + y \geq 100$

Determining Solutions to a System of Linear Inequalities Algebraically

In a system of linear inequalities, the inequalities are known as constraints because the values of the expressions are "constrained" to lie within a certain region. To test a point as a possible solution to a system of linear inequalities, substitute the x- and y-values into each inequality. If both inequalities are true for the given values, the point is a solution of the system. If the values make only one or neither inequality true, the point is not a solution.

Example

| Point	$2x + y > 4$	$x - y \leq 3$	Description of solution
$(-2, -6)$	$-4 + (-6) > 4$ $-10 > 4$ ✗	$-2 - (-6) \leq 3$ $4 \leq 3$ ✗	The point is not a solution to either inequality so it is not a solution of the system of inequalities.
$(2, 6)$	$4 + 6 > 4$ $10 > 4$ ✓	$2 - 6 \leq 3$ $-4 \leq 3$ ✓	The point is a solution to both inequalities so it is a solution of the system of inequalities.
$(2, -1)$	$4 + (-1) > 4$ $3 > 4$ ✗	$2 - (-1) \leq 3$ $3 \leq 3$ ✓	The point is a solution to the second inequality but not the first so it is not a solution of the system of inequalities.

7

7.2 Graphing a System of Linear Inequalities

The solution of a system of linear inequalities is the intersection of the solutions of each inequality. Every point in the intersection region satisfies the system. Graph each inequality on the same coordinate plane. The region that overlaps is the solution to the system.

Example

$$\begin{cases} 2x + y \leq 7 \\ x > 3 \end{cases}$$

Two points that are solutions of the system are $(4, -5)$ and $(5, -8)$.
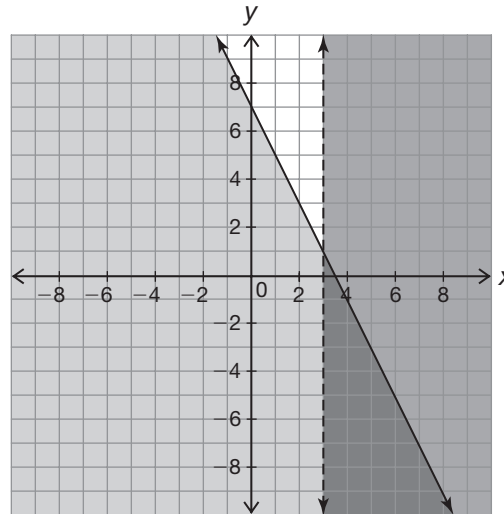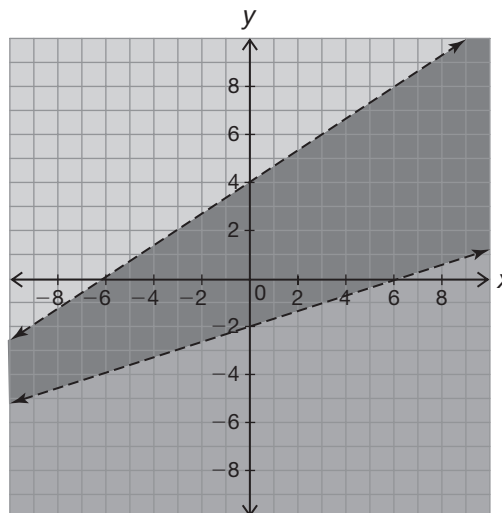

7.2 Solving a System of Linear Inequalities Using a Graphing Calculator

The solution of a system of linear inequalities is the intersection of the solutions of each inequality. Every point in the intersection region satisfies the system. Use a graphing calculator to graph each inequality on the same coordinate plane. The region that overlaps is the solution of the system. Use the **value** function to identify solutions to the system of inequalities.

Example

$$\begin{cases} y < \dfrac{2}{3}x + 4 \\ y > \dfrac{1}{3}x - 2 \end{cases}$$

Two points that are solutions to the system are $(4, 4)$ and $(-2, 1)$.

7

7.3 Writing a System of More Than Two Inequalities

Systems of inequalities can consist of more than two linear inequalities. The inequalities can be written to represent the constraints of a problem situation.

Example

Colby must sell at least 50 items in his school's fundraiser to earn a prize. To pay for his school trip, Colby must make at least $200. Trail mix sells for $2 a bag and magazine subscriptions sell for $8 each.

Let t represent the number of bags of trail mix sold and let m represent the number of magazine subscriptions sold.

$$\begin{cases} t + m \geq 50 \\ 2t + 8m \geq 200 \\ t \geq 0 \\ m \geq 0 \end{cases}$$

7.3 Solving a System of More Than Two Inequalities by Graphing

To solve a system of linear inequalities with more than two inequalities, graph each linear inequality and shade the correct region that contains the solution sets that satisfy all the inequalities in the system. Also, determine all the points of intersection for the boundary lines that make the vertices of the solution region. Use a closed point if the point is part of the solution region, and use an open dot if the point is not part of the solution region, but is a point of intersection.

Example

$$\begin{cases} x \geq -4 \\ y > -2 \\ y \leq 6 \\ y > 4x + 3 \end{cases}$$

A solution of the system of inequalities is $(-2, 2)$.

7
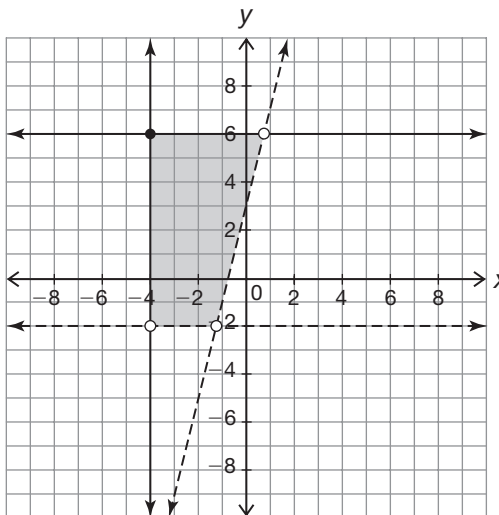
7.4 Identifying the Maximum and Minimum Values of a Linear Expression

Linear programming is a branch of mathematics that determines the maximum and minimum value of linear expressions on a region produced by a system of linear inequalities. Use the given information to define your variables and identify the constraints as a system of linear inequalities. Graph the solution set for the system of linear inequalities and shade the region that contains the solution set. Label all points of intersection of the boundary lines. The maximum and minimum values of a system of inequalities occur at a vertex of the region defined by the system. To determine the maximum and minimum, substitute each point into a given function.

Example

Tony mows lawns and trims shrubbery to make extra money. It takes him 30 minutes to mow an average lawn and 60 minutes to trim shrubbery. Tony can spend no more than 20 hours a week on his landscaping business because of school. He can afford to buy enough gas for the lawnmower to mow no more than 14 lawns a week. After the cost of gas, Tony makes a profit of $20 for each yard mowed and $15 for each yard in which he trims the shrubbery.

Let x represent the number of lawns Tony mows per week and y represent the number of yards for which Tony trims shrubbery.

$$\begin{cases} 0.5x + y \leq 20 \\ x \leq 14 \\ x \geq 0 \\ y \geq 0 \end{cases}$$

$P(x, y) = 20x + 15y$
$P(0, 0) = 20(0) + 15(0) = 0$
$P(0, 20) = 20(0) + 15(20) = 300$
$P(14, 0) = 20(14) + 15(0) = 280$
$P(14, 13) = 20(14) + 15(13) = 475$

To maximize his profit, Tony should mow 14 lawns and trim shrubbery in 13 yards each week. His maximum profit would be $475.

7

7